PETRI NET BASED
TRAJECTORY OPTIMIZATION

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1. Introduction
2. The CEGAR approach on Petri nets
3. Trajectory optimization using CEGAR
4. Evaluation
5. Conclusions
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**INTRODUCTION**

Petri Nets

- **Information systems are becoming more complex**
  - Modeling and automatic analysis is important

- **Modeling: Petri Nets**
  - Widely used modeling formalism
    - Asynchronous, distributed, parallel, non-deterministic systems
  - Behavior: possible states and transitions
  - Optimization problems
    - Optimal trajectory from the initial state to a given goal state
    - Reachability analysis
INTRODUCTION

Reachability analysis

• Reachability analysis
  – Checks, if a given state is reachable from the initial state
  – \( m_1 \in R(PN, m_0) \Rightarrow \text{"Is } m_1 \text{ reachable from } m_0 \text{ in the Petri net } PN?" \)
  – Drawback: complexity

• Complexity
  – State space can be large or infinite
  – Reachability is decidable, but at least EXPSPACE-hard
  – No upper bound is known
  – A possible solution is to use abstraction
INTRODUCTION
The CEGAR approach

- **CounterExample Guided Abstraction Refinement**
  - General approach
    - Can handle large or infinite state spaces
  - Works on an abstraction of the original model
    - Less detailed state space
    - Finite, smaller representation
  - Abstraction refinement is required
    - An action in the abstract model may not be realizable in the original model
    - Refine the abstraction using the information from the explored part of the state space
  - H. Wimmel, K. Wolf
    - Applying CEGAR to the Petri Net State Equation (2011)
OUTLINE OF THE TALK

1. Introduction

2. The CEGAR approach on Petri nets

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• **Abstraction of Petri nets: state equation**

\[ m_0 + Cx = m_1 \]

- **Initial state**
- **Target state**

**Incidence matrix**

\[
\begin{bmatrix}
2 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & 1 & 1 & -1 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 1
\end{bmatrix}
\]

**Firing count of transitions** *(unknown)*
CEGAR APPROACH ON PETRI NETS
Analysis of the abstract model

- Solving the state equation for the firing count of transitions
- Integer Linear Programming problem
- Necessary, but not sufficient criterion for reachability

\[ m_0 + Cx = m_1 \]
CEGAR APPROACH ON PETRI NETS

Examining the solution

- Bounded exploration of the state space
  - Try to fire the transitions in some order

Analysis of the abstract model

No solution

Not reachable

Solution

Reachable

Realizable

Reachability problem

Initial abstraction

State equation

Examine the solution
CEGAR APPROACH ON PETRI NETS
Abstraction refinement

- Exclude the counterexample without losing any realizable solution
- Constraints can be added to the state equation
  - The state equation may become infeasible
  - A new solution can be obtained
- Traversing the solution space instead of the state space
CEGAR APPROACH ON PETRI NETS

Solution space

- **Semi-linear space**
  - Base solutions
  - T-invariants
    - Solutions of the homogenous part $Cy = 0$
    - Possible cycles in the Petri Net

- **Two types of constraints**
  - Jump: switch between base solutions
  - Increment: reach non-base solutions

$\mathbf{m}_0 + \mathbf{Cx} = \mathbf{m}_1$
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TRAJECTORY OPTIMIZATION
Extensions to the CEGAR approach

• Our previous work
  – Analyzing the algorithm
    • Correctness
    • Completeness
  – Extending the set of decidable problems
  – New optimizations

• Current work
  – Trajectory optimization using CEGAR
    • Assigning costs to transitions
    • New strategy for the solution space traversal
TRAJECTORY OPTIMIZATION
Assigning costs to transitions

- **Core of the CEGAR approach: state equation**
  - ILP problem
  - ILP solver minimizes a function over the variables
  - Variables are transitions in our case

- **Original algorithm**
  - Verification purpose → *Is there a solution or not?*
  - Equal cost for each transition → shortest trajectories

- **Our new approach**
  - Optimization purpose → *What is the optimal solution?*
  - Arbitrary cost for transitions
  - ILP solver minimizes using the given cost
• **Traversing the solution space of the state equation**

• **Original algorithm**
  – Verification purpose → *Is there a solution or not?*
  – Fast convergence → DFS

• **Our new approach**
  – Optimization purpose → *What is the optimal solution?*
  – Store the solutions in a sorted queue
  – Continue with the one with the lowest cost
Input: Reachability problem $m_1 \in R(PN, m_0)$ and cost function $z$
Output: Trajectory $\sigma$ or „Not reachable”

1. $C \leftarrow$ incidence matrix of $PN$
2. $Q \leftarrow$ SolveILP($m_0$, $m_1$, $C$, $z$, $\emptyset$)
3. while $Q \neq \emptyset$ do
4. | $x \leftarrow$ solution from $Q$ minimizing $z \cdot x$
5. | if $x$ is realizable then stop and output $\sigma$ for $x$
6. | else
7. | | foreach jump and increment constraint $c'$ do
8. | | | $Q \leftarrow$ SolveILP($m_0$, $m_1$, $C$, $z$, $\{\text{constraints of } x\} \cup \{c'\}$)
9. | | end foreach
10. | | end else
11. | end while
12. Output „Not reachable”

Initial abstraction
Analysis of the abstract model
Examine solution
Rechable
Analysis of the abstract model
Refine abstraction
Not reachable
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EVALUATION

• **Implementation**
  – PetriDotNet framework
    • Modeling and analysis of Petri nets
    • Supports add-ins

• **Measurements**
  – Traveling salesman problem
    • Graph traversal optimization
    • NP-complete

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CONCLUSIONS

• **New approach for the optimal trajectory problem**
  – Translation to the reachability of Petri nets
  – Solving reachability using CEGAR
    • Handle transition costs
    • New strategy for solution space traversal

• **Implementation and evaluation**

• **Possible future direction**
  – Optimization of continuous systems
TÁMOP-4.2.2.C-11/1/KONV-2012-0004
National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

THANK YOU FOR YOUR ATTENTION!

QUESTIONS?

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