NATIONAL COORDINATING CENTER FOR INFOCOMMUNICATIONS

TÁMOP-4.2.2.C-11/1/KONV-2012-0004

National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

PETRI NET BASED TRAJECTORY OPTIMIZATION

Ákos Hajdu, Róbert Németh,
Szilvia Varró-Gyapay, András Vörös

VOCAL 2014, Veszprém, Hungary, 16.12.2014.

SZÉCHENYI 2020

HUNGARIAN GOVERNMENT

European Union
European Social
Fund

INVESTING IN YOUR FUTURE

OUTLINE OF THE TALK

TÁMOP-4.2.2.C-11/1/KONV-2012-0004

National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

1. Introduction
2. The CEGAR approach on Petri nets
3. Trajectory optimization using CEGAR
4. Evaluation
5. Conclusions

OUTLINE OF THE TALK

TÁMOP-4.2.2.C-11/1/KONV-2012-0004

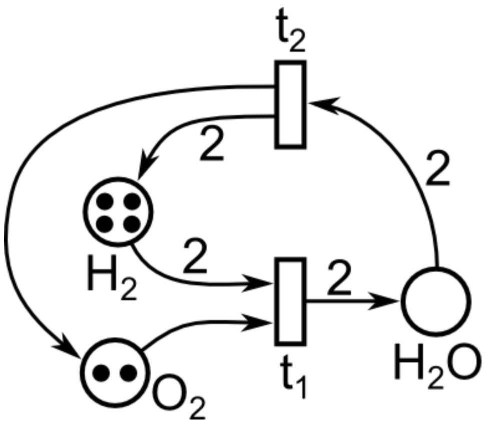
National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

1. **Introduction**
2. The CEGAR approach on Petri nets
3. Trajectory optimization using CEGAR
4. Evaluation
5. Conclusions

TÁMOP-4.2.2.C-11/1/KONV-2012-0004
National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

- **Information systems are becoming more complex**
  - Modeling and automatic analysis is important

- **Modeling: Petri Nets**
  - Widely used modeling formalism
    - Asynchronous, distributed, parallel, non-deterministic systems
  - Behavior: possible states and transitions
  - Optimization problems
    - Optimal trajectory from the initial state to a given goal state
    - Reachability analysis



TÁMOP-4.2.2.C-11/1/KONV-2012-0004

National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

- **Reachability analysis**
  - Checks, if a given state is reachable from the initial state
  - $m_1 \in R(PN, m_0)$ → *„Is m_1 reachable from m_0 in the Petri net PN?"*
  - Drawback: complexity
- **Complexity**
  - State space can be large or infinite
  - Reachability is decidable, but at least EXPSPACE-hard
  - No upper bound is known
  - A possible solution is to use abstraction

INTRODUCTION
The CEGAR approach

- **CounterExample Guided Abstraction Refinement**
  - General approach
    - Can handle large or infinite state spaces
  - Works on an abstraction of the original model
    - Less detailed state space
    - Finite, smaller representation
  - Abstraction refinement is required
    - An action in the abstract model may not be realizable in the original model
    - Refine the abstraction using the information from the explored part of the state space
  - H. Wimmel, K. Wolf
    - Applying CEGAR to the Petri Net State Equation (2011)

OUTLINE OF THE TALK

1. Introduction
2. **The CEGAR approach on Petri nets**
3. Trajectory optimization using CEGAR
4. Evaluation
5. Conclusions

CEGAR APPROACH ON PETRI NETS

Initial abstraction

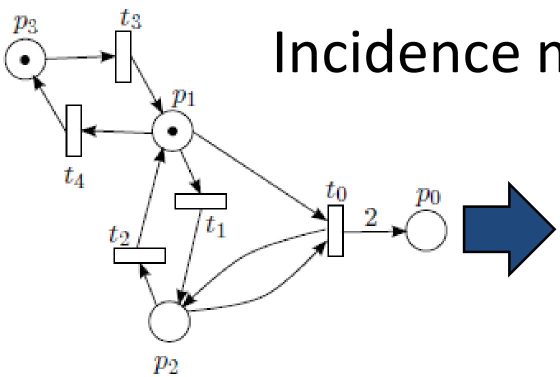
- **Abstraction of Petri nets: state equation**

$$\mathbf{m_0 + Cx = m_1}$$

Initial state

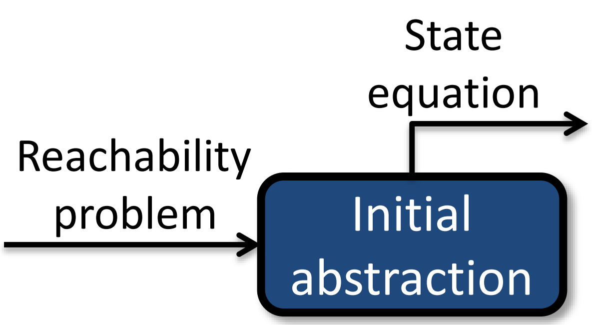
Target state

Incidence matrix



$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 1 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Firing count of transitions *(unknown)*

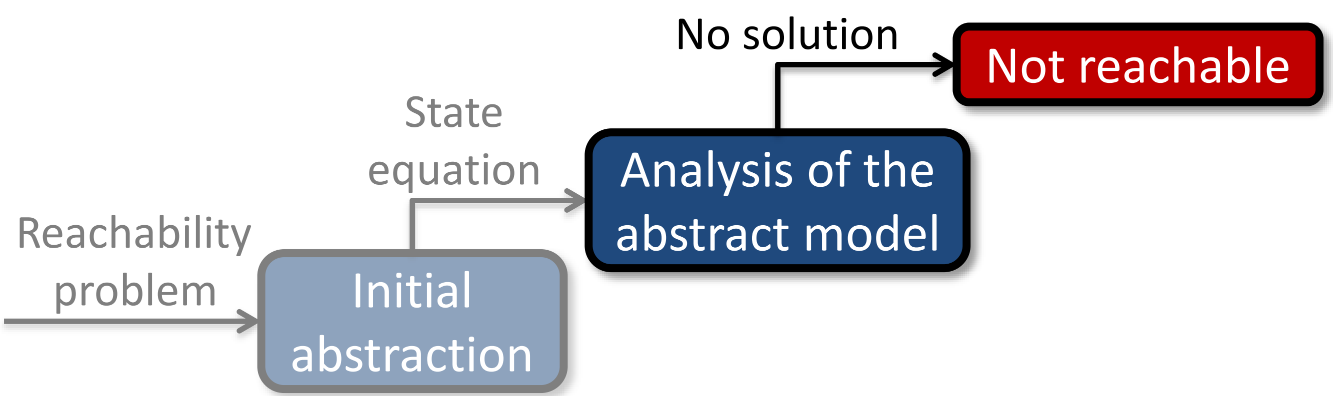


CEGAR APPROACH ON PETRI NETS

Analysis of the abstract model

- **Solving the state equation for the firing count of transitions**
- **Integer Linear Programming problem**
- **Necessary, but not sufficient criterion for reachability**

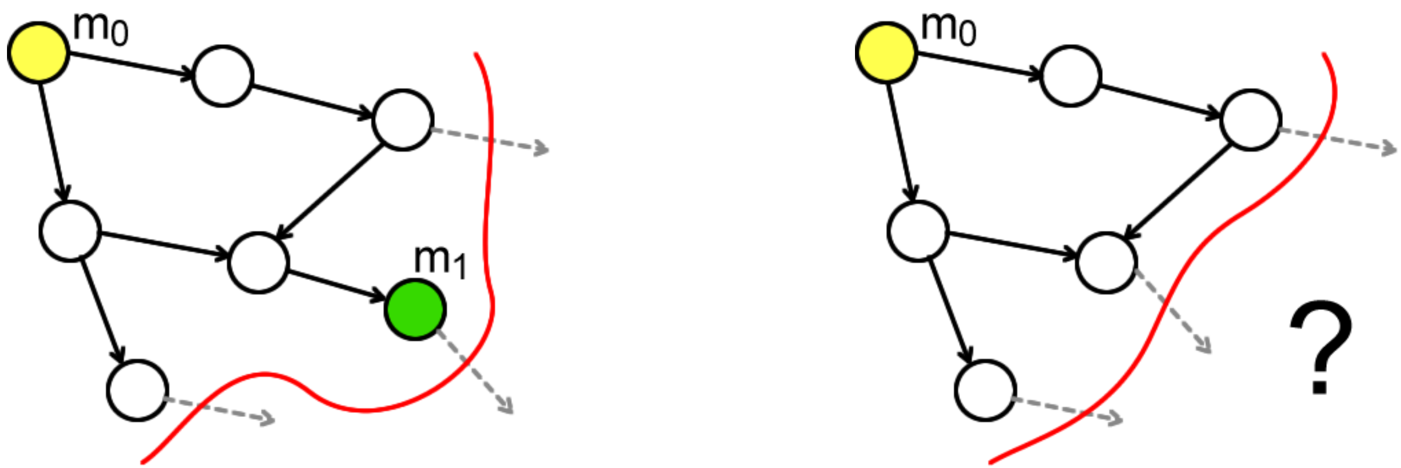
$$m_0 + C\mathbf{x} = m_1$$

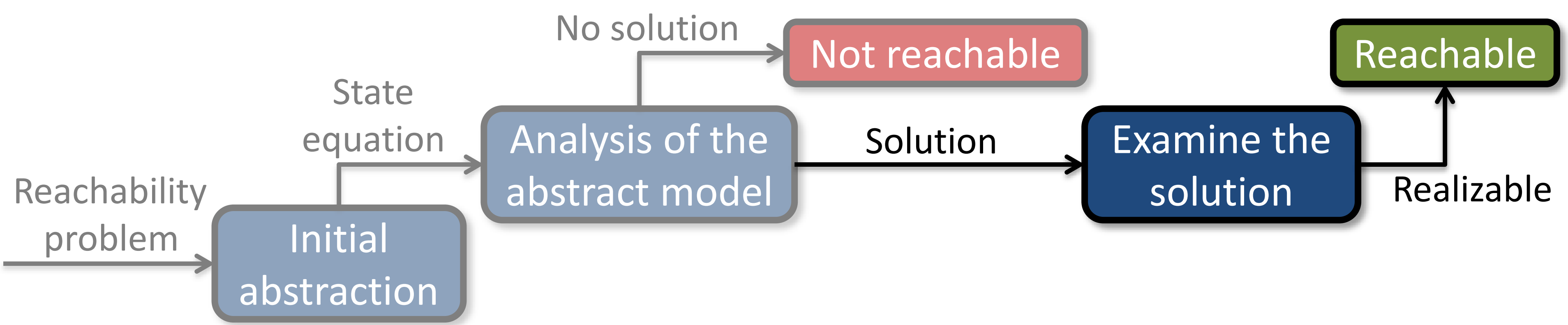


CEGAR APPROACH ON PETRI NETS

Examining the solution

- **Bounded exploration of the state space**
  - Try to fire the transitions in some order

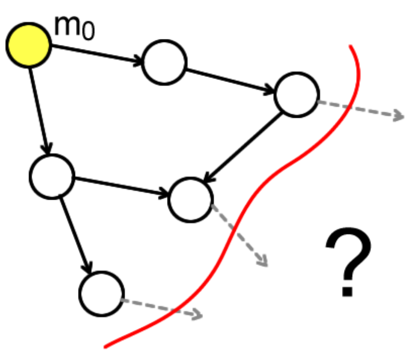


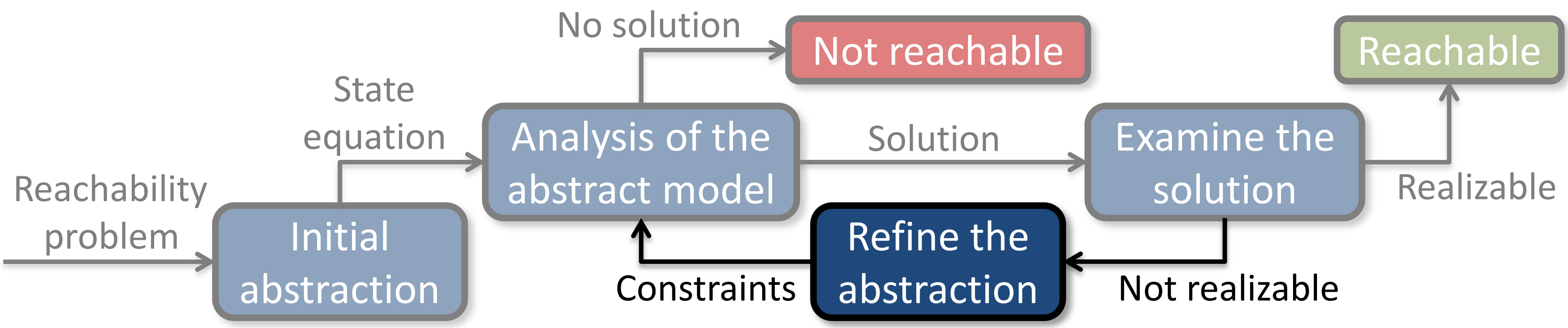


CEGAR APPROACH ON PETRI NETS

Abstraction refinement

- **Exclude the counterexample without losing any realizable solution**
- **Constraints can be added to the state equation**
  - The state equation may become infeasible
  - A new solution can be obtained
- **Traversing the solution space instead of the state space**



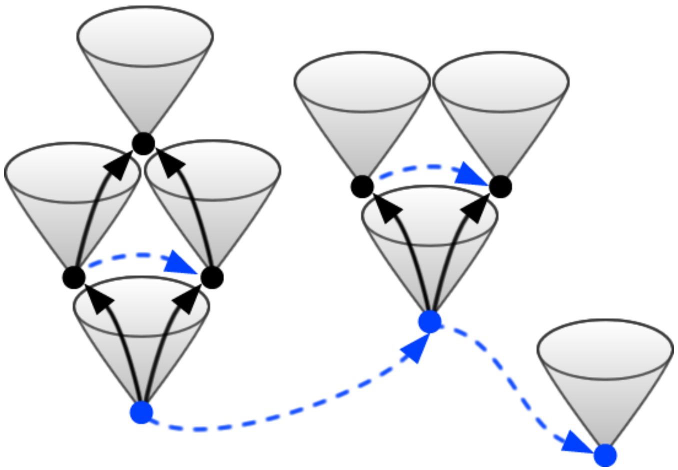


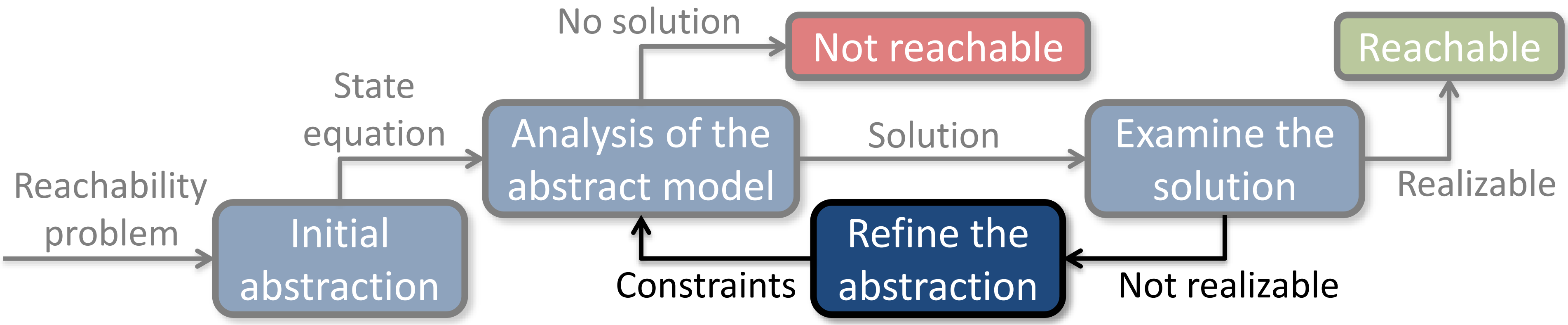
TÁMOP-4.2.2.C-11/1/KONV-2012-0004

National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

$$m_0 + C\mathbf{x} = m_1$$

- **Semi-linear space**
  - Base solutions
  - T-invariants
    - Solutions of the homogenous part Cy = 0
    - Possible cycles in the Petri Net
- **Two types of constraints**
  - Jump: switch between base solutions
  - Increment: reach non-base solutions





OUTLINE OF THE TALK

1. Introduction
2. The CEGAR approach on Petri nets
3. **Trajectory optimization using CEGAR**
4. Evaluation
5. Conclusions

TÁMOP-4.2.2.C-11/1/KONV-2012-0004

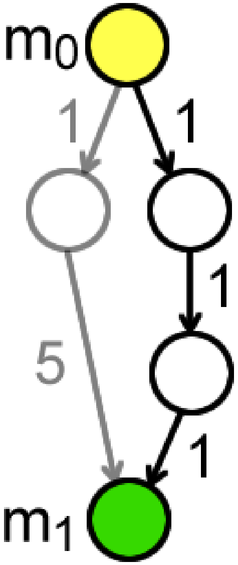
National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

- **Our previous work**
  - Analyzing the algorithm
    - Correctness
    - Completeness
  - Extending the set of decidable problems
  - New optimizations
- **Current work**
  - Trajectory optimization using CEGAR
    - Assigning costs to transitions
    - New strategy for the solution space traversal

TRAJECTORY OPTIMIZATION

Assigning costs to transitions

- **Core of the CEGAR approach: state equation**
  - ILP problem
  - ILP solver minimizes a function over the variables
  - Variables are transitions in our case
- **Original algorithm**
  - Verification purpose → *Is there a soluton or not?*
  - Equal cost for each transition → shortest trajectories
- **Our new approach**
  - Optimization purpose → *What is the optimal solution?*
  - Arbitrary cost for transitions
  - ILP solver minimizes using the given cost



TRAJECTORY OPTIMIZATION

New solution space traversal strategy

- **Traversing the solution space of the state equation**
- **Original algorithm**
  - Verification purpose → *Is there a soluton or not?*
  - Fast convergence → DFS
- **Our new approach**
  - Optimization purpose → *What is the optimal solution?*
  - Store the solutions in a sorted queue
  - Continue with the one with the lowest cost

TÁMOP-4.2.2.C-11/1/KONV-2012-0004
National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

PSEUDO CODE

Input: Reachability problem $m_1 \in R(PN, m_0)$ and cost function z

Output: Trajectory σ or „*Not reachable*"

```
C ← incidence matrix of PN
Q ← SolveILP(m0, m1, C, z, ∅)
while Q ≠ ∅ do
|   x ← solution from Q minimizing z·x
|   if x is realizable then stop and output σ for x
|   else
|   |   foreach jump and increment constraint c' do
|   |   |   Q ← SolveILP(m0, m1, C, z, {constraints of x} ∪ {c'})
|   |   end foreach
|   end else
end while
Output „Not reachable"
```

Initial abstraction
Analysis of the abstract model
Rechable
Examine solution
Analysis of the abstract model
Refine abstraction
Not reachable

OUTLINE OF THE TALK

TÁMOP-4.2.2.C-11/1/KONV-2012-0004

National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

1. Introduction
2. The CEGAR approach on Petri nets
3. Trajectory optimization using CEGAR
4. **Evaluation**
5. Conclusions

EVALUATION

- **Implementation**
  - PetriDotNet framework
    - Modeling and analysis of Petri nets
    - Supports add-ins
- **Measurements**
  - Traveling salesman problem
    - Graph traversal optimization
    - NP-complete

Number of nodes	Runtime (s)
4	0,04
6	0,14
8	0,66
9	0,90
10	1,95
11	9,49
12	24,57
13	1067,00

OUTLINE OF THE TALK

TÁMOP-4.2.2.C-11/1/KONV-2012-0004
National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

1. Introduction
2. The CEGAR approach on Petri nets
3. Trajectory optimization using CEGAR
4. Evaluation
5. **Conclusions**

CONCLUSIONS

- **New approach for the optimal trajectory problem**
  - Translation to the reachability of Petri nets
  - Solving reachability using CEGAR
    - Handle transition costs
    - New strategy for solution space traversal
- **Implementation and evaluation**
- **Possible future direction**
  - Optimization of continuous systems

TÁMOP-4.2.2.C-11/1/KONV-2012-0004

National Research Center for Development and Market Introduction of Advanced Information and Communication Technologies

THANK YOU FOR YOUR ATTENTION!

QUESTIONS?

hajduakos182@gmail.com; vori@mit.bme.hu

https://inf.mit.bme.hu/en/research/tools/petridotnet

European Union
European Social Fund
INVESTING IN YOUR FUTURE